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Do 10 steps. solve exactly. compute
the error. Use 6 decimal places.

$$y' = y - y^2; \quad y(0) = 0.2, \quad h = 0.1$$

exact solution:

$$\frac{dy}{y - y^2} = dx$$

$$\frac{dy}{(y - \frac{1}{2})^2 - \frac{1}{4}} = -dx$$

$$\int \frac{dy}{(y - \frac{1}{2})^2 - \frac{1}{4}} = -x + C$$

$$u = y - \frac{1}{2}$$

$$du = dy$$

$$\int \frac{du}{u^2 - \frac{1}{4}} = -x + C$$

using the list of integrals of rational functions:

$$-2 \operatorname{arctanh}(2u) = -x + C \quad \left(\begin{array}{l} (-\frac{1}{a}) \operatorname{arctanh}(\frac{x}{a}) \\ \text{at } |x| < |a| \end{array} \right)$$

$$2u = \operatorname{arctanh}\left(\frac{x}{2}\right) + C$$

$$\text{if } y - \frac{1}{2} > 0 \Rightarrow y > \frac{1}{2}$$

$$y - \frac{1}{2} < \frac{1}{2}$$

$$y < 1$$

$$\text{if } y < \frac{1}{2}$$

$$-(y - \frac{1}{2}) < \frac{1}{2}$$

$$y - \frac{1}{2} > -\frac{1}{2}$$

$$y > 0$$

The bounds for

the possible values

of y where the

function

is correct is

$$0 < y < 1$$

(12-16) correct dimensions

$$2y - \frac{1}{2} = \tanh\left(\frac{x}{2}\right) + C$$

$$2y = \tanh\left(\frac{x}{2}\right) + C$$

$$y = \frac{\tanh\left(\frac{x}{2}\right)}{2} + C$$

applying initial conditions?

$$0.2 = \frac{\tanh\left(\frac{0}{2}\right)}{2} + C$$

$$\therefore C = 0.2$$

$$y = \frac{\tanh\left(\frac{x}{2}\right)}{2} + 0.2$$

David Kennedy Villanueva
 before we find the approximate solution,
 let us find where $1 - 0.646^x = 0$!

$$1 - 0.646^x = 0$$

$$e^x = \frac{1}{0.646} \quad \text{or } x =$$

$$x = -\ln(0.646)$$

$$x \approx 0.446287$$

The formula that would be used is
 as follows:

$$y_1 = y_0 + f(x_0, y_0) h$$

h is the step size

This formula is valid for equations of the
 form $\frac{dy}{dx} = P(x, y)$ with initial conditions $y(x_0) = y_0$

For example in our equation $\frac{dy}{dx} = y - y^2, y(0) = 0.2, h = 0.1$

Index	x	y
0	0	0.2
1	0.1	$0.2 + (0.2 - 0.2^2)(0.1)$

$h = 0.1$

Daniel Kanneb's Villagers

Index (step)	x	y	exact value	error
0	0	0.2	0.2	0
1	0.1	0.216	0.224979	0.008879
2	0.2	0.232934	0.245834	0.0169
3	0.3	0.250802	0.274943	0.023641
4	0.4	0.269592	0.298688	0.029096
5	0.5	0.289283	0.322459	0.033176
6	0.6	0.309843	0.345656	0.035813
7	0.7	0.331227	0.368188	0.036961
8	0.8	0.353379	0.389974	0.036596
9	0.9	0.376229	0.41095	0.034721
10	1	0.399698	0.431055	0.031362